

# Search Strategies for Rectangle Packing

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# Outline

- 1 Introduction
- 2 Model
- 3 Search Strategies
- 4 Model Improvements
- 5 Results
- 6 Extensions

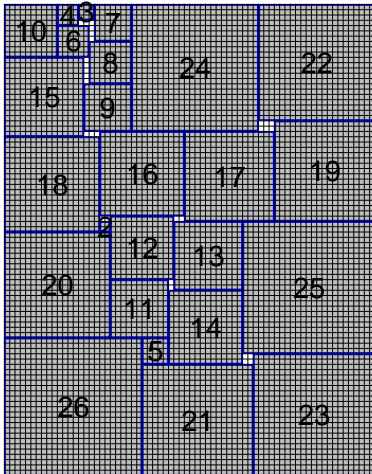
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# Overview

- Use of off-the-shelf constraint technology
- SICStus Prolog 4.02
- Evaluate different search strategies
- Problem: Packing all squares from  $1 \times 1$  to  $n \times n$  in smallest rectangle
- Difficulty: Aspect ratio of enclosing rectangle not known

## Problem (N=26)



# Motivation

- Related to large-scale, real-life problems
  - IC floor planning, PCB design, architectural design
- Mix of feasible and infeasible subproblems
- Amount of slack changes with subproblem
- New results for open problems
  - Packing in Rectangle ( $N=26, 27$ )
  - Packing in Square ( $N=26, 27, 29, 30, 31, 35$ )

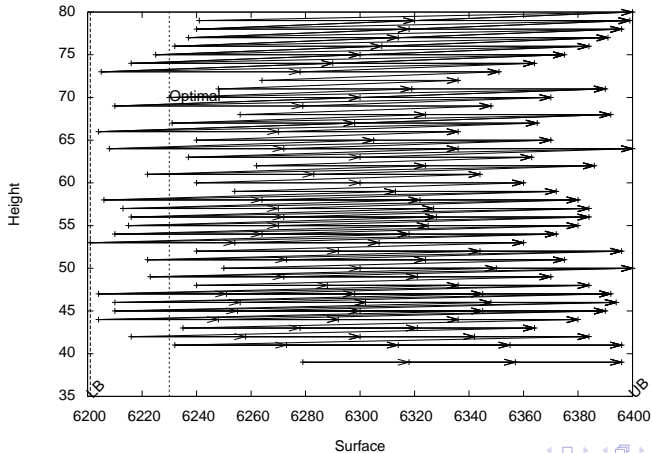
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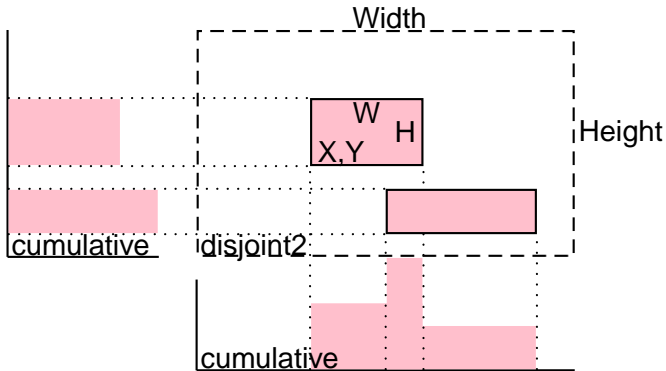
## Problem Decomposition

- Search for candidate enclosing rectangle
- Area must be larger than sum of items to be placed
- Search in order of increasing area
  - and increasing “squareness”
- Check each candidate for (in)feasibility until first solution is found
- Observation: Only limited number of candidates explored

# Candidates



# Basic Model



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# Alternatives

- naive
- x then y
- disjunctive
- semantic disjunctive
- dual
- interval
- split
- xy interval

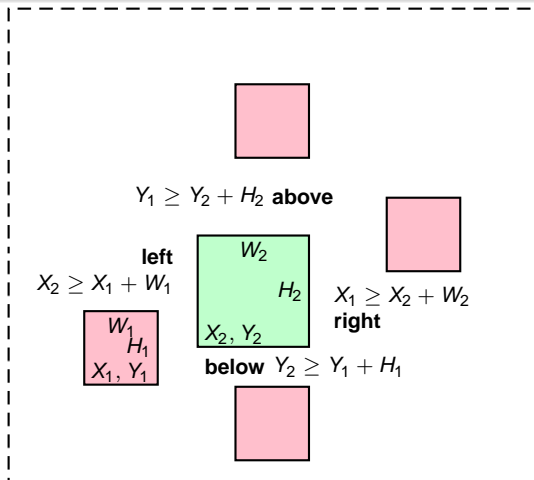
# Naive

- Place items in order of decreasing size
- Fix  $X$  and  $Y$  value for each item
- Depth-first search to explore search space
- Problem: Large number of alternatives considered

## X then Y

- Fix  $X$  value for all items, before assigning  $Y$  values
- Intuition: Once all  $X$  values are fixed, the  $Y$  values are very constrained
- Ideal: Search tree  $N$ , not  $2 \times N$  levels deep
- Problem: May lead to deep thrashing if propagation too weak

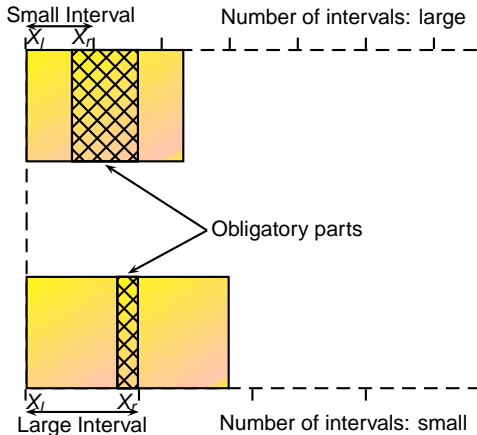
# Disjunctive



## Interval Based Strategies

- Key Idea: Fixing intervals, not values
- Fixing variables to values is too restrictive
- Select “area” in which item is placed
- Allows items to shift slightly
- Restrict domain to intervals
- Only at end fix actual values

# Forcing Obligatory Parts



# Variants

- (X) Interval
  - Split all  $X$  variables into intervals
  - Then fix  $X$  values
  - Then treat all  $Y$  variables the same way
- Split
  - Split  $X$  variables into intervals
  - Split  $Y$  variables into intervals
  - Then fix values
- XY Interval
  - For each item, split  $X$  and  $Y$  variables into intervals
  - Then fix values

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## Ignore $1 \times 1$ items

- They can be placed anywhere
- No need to include in propagation
- Interact with search routine, creating useless branches

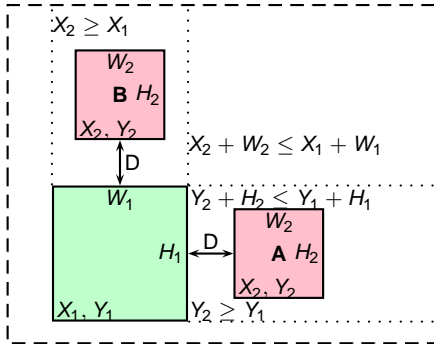
## Adaptation of Korf's Dominance Criterion

- Certain placements are dominated by others
- No need to explore both
- Two variants
  - Ignore placement close to the border
  - Interaction of two items

## Forbidden Gaps due to Dominance

|          |   |   |   |     |      |       |       |       |       |       |    |
|----------|---|---|---|-----|------|-------|-------|-------|-------|-------|----|
| size     | 2 | 3 | 4 | 5-8 | 9-11 | 12-17 | 18-21 | 22-29 | 30-34 | 34-44 | 45 |
| generic  | 1 | 2 | 2 | 3   | 4    | 5     | 6     | 7     | 8     | 9     | 10 |
| specific | 2 | 3 |   |     |      |       |       |       |       |       |    |

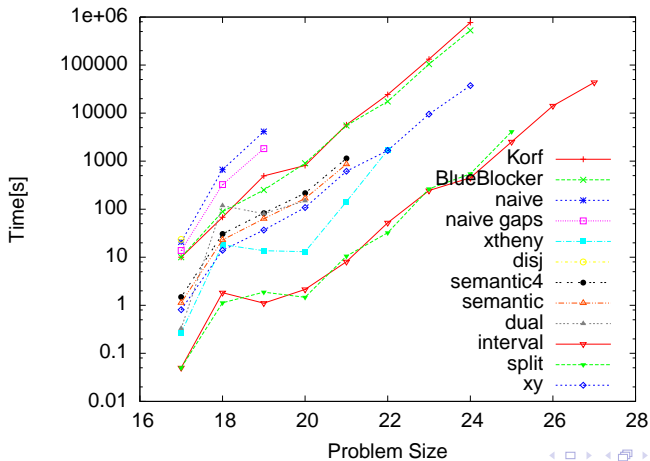
# Dominance Criterion



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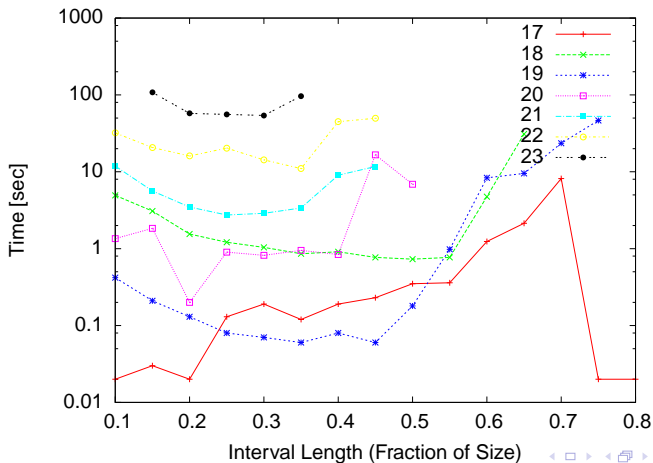
# Strategies Comparison



# Strategy Comparison

| N  | naive   | xtheny  | disj  | semantic4 | semantic | dual   | interval<br>0.3 | split<br>0.2 | xy<br>0.75 |
|----|---------|---------|-------|-----------|----------|--------|-----------------|--------------|------------|
| 15 | 2.92    | 0.09    | 12.12 | 0.55      | 0.45     | 2.63   | -               | 0.05         | -          |
| 16 | 10.44   | 0.11    | 98.25 | 1.31      | 1.03     | 0.89   | -               | 0.05         | -          |
| 17 | 20.75   | 0.27    | 23.57 | 1.48      | 1.13     | 0.33   | 0.05            | 0.05         | 0.81       |
| 18 | 667.33  | 18.37   | -     | 30.53     | 23.05    | 118.58 | 1.83            | 1.13         | 13.94      |
| 19 | 4140.09 | 13.73   | -     | 83.42     | 63.25    | 80.66  | 1.11            | 1.88         | 36.78      |
| 20 | -       | 13.08   | -     | 216.07    | 167.61   | 149.79 | 2.14            | 1.47         | 108.28     |
| 21 | -       | 143.72  | -     | 1138.98   | 865.13   | -      | 8.09            | 10.59        | 619.45     |
| 22 | -       | 1708.89 | -     | -         | -        | -      | 52.21           | 32.36        | 1668.59    |
| 23 | -       | -       | -     | -         | -        | -      | 245.07          | 265.54       | 9521.73    |
| 24 | -       | -       | -     | -         | -        | -      | 452.73          | 545.82       | 37506.20   |
| 25 | -       | -       | -     | -         | -        | -      | 2533.64         | 4127.41      | -          |
| 26 | -       | -       | -     | -         | -        | -      | 14158.15        | -            | -          |
| 27 | -       | -       | -     | -         | -        | -      | 43529.87        | -            | -          |

# Impact of Interval Length



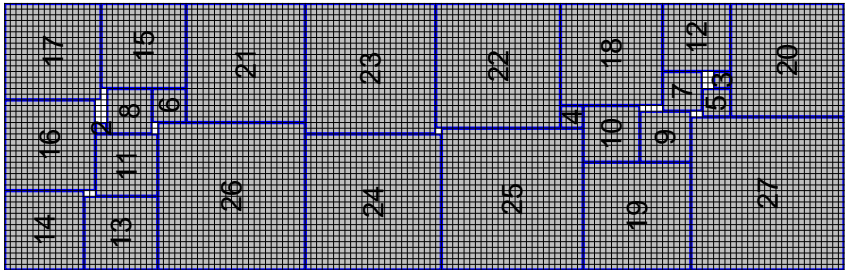
## Method Comparison

| N  | pure   | gap    | domain | notone | all   | best  |
|----|--------|--------|--------|--------|-------|-------|
| 18 | 100.00 | 99.37  | 78.96  | 12.93  | 9.77  | 9.78  |
| 19 | 100.00 | 101.61 | 87.14  | 48.55  | 38.26 | 37.31 |
| 20 | 100.00 | 105.26 | 92.24  | 18.93  | 16.20 | 15.39 |
| 21 | 100.00 | 100.94 | 81.90  | 63.57  | 50.82 | 49.58 |
| 22 | 100.00 | 100.24 | 90.56  | 23.66  | 19.46 | 19.00 |
| 23 | 100.00 | 99.81  | 78.92  | 30.33  | 23.18 | 22.80 |
| 24 | 100.00 | 101.77 | 77.69  | 36.43  | 29.16 | 28.58 |

# Rectangle Placement Overview

| N  | Surface | K  | Width | Height | Area | Loss | Time     | Clautiaux | Korf      | BlueBlocker |
|----|---------|----|-------|--------|------|------|----------|-----------|-----------|-------------|
| 18 | 2109    | 14 | 31    | 69     | 2139 | 1.42 | 00:01    | 31:33     | 1:08      | 1:29        |
| 19 | 2470    | 12 | 47    | 53     | 2491 | 0.85 | 00:01    | 72:53:18  | 8:15      | 4:11        |
| 20 | 2870    | 14 | 34    | 85     | 2890 | 0.70 | 00:02    | -         | 13:32     | 15:03       |
| 21 | 3311    | 19 | 38    | 88     | 3344 | 1.00 | 00:07    | -         | 1:35:08   | 1:32:01     |
| 22 | 3795    | 15 | 39    | 98     | 3822 | 0.71 | 00:51    | -         | 6:46:15   | 4:51:23     |
| 23 | 4324    | 19 | 64    | 68     | 4352 | 0.65 | 03:58    | -         | 36:54:50  | 29:03:49    |
| 24 | 4900    | 18 | 56    | 88     | 4928 | 0.57 | 05:56    | -         | 213:33:00 | 146:38:48   |
| 25 | 5525    | 17 | 43    | 129    | 5547 | 0.40 | 40:38    | -         | see paper | -           |
| 26 | 6201    | 21 | 70    | 89     | 6230 | 0.47 | 03:41:43 | -         | -         | -           |
| 27 | 6930    | 21 | 47    | 148    | 6956 | 0.38 | 11:30:02 | -         | -         | -           |

# Optimal Solution (N=27)



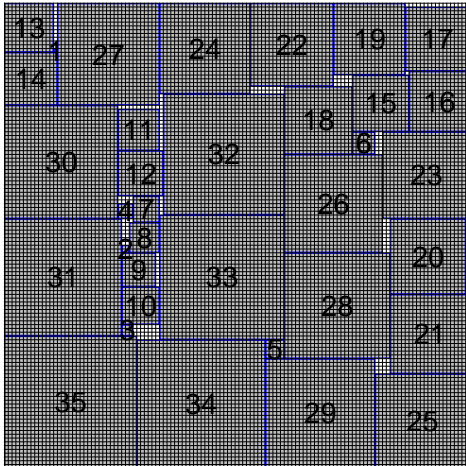
# Packing In Square

- Pack all squares  $1 \times 1$  to  $N \times N$  into smallest square
- Fewer candidates to check, start with lower bound  
$$\lceil \sqrt{\sum_{i=1}^N i^2} \rceil$$
- Often, lower bound is reached  $\implies$  optimal
- More slack in optimal solution than for rectangle packing
- Improved symmetry breaking possible

# Optimal Solution for Packing in Square

|                    |         |       |       |      |       |         |
|--------------------|---------|-------|-------|------|-------|---------|
| Problem Size       | 26      | 27    | 29    | 30   | 31    | 35      |
| Optimal Solution   | 80      | 84    | 93    | 98   | 103   | 123     |
| $T_{\text{opt}}$   | 12:26   | 00:04 | 11:06 | 2:07 | 00:18 | 1:10:07 |
| $T_{\text{proof}}$ | 1:25:22 | -     | -     | -    | -     | -       |

# Optimal Solution (N=35)



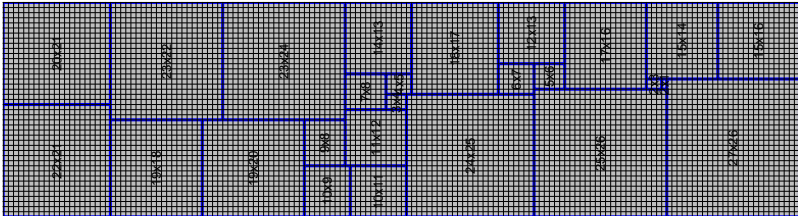
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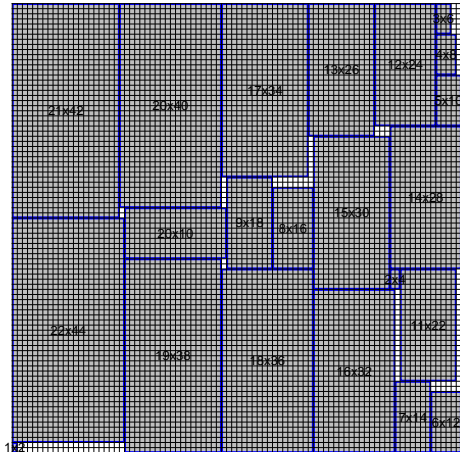
# Packing Rectangles, not Squares

- Rectangles may or may not be rotated
- Introduces new degree of freedom
- Does not change constraint model
- Rotation handled as part of search
- Surprising: impact quite small

# Almost Square, Optimal Solution (N=26)



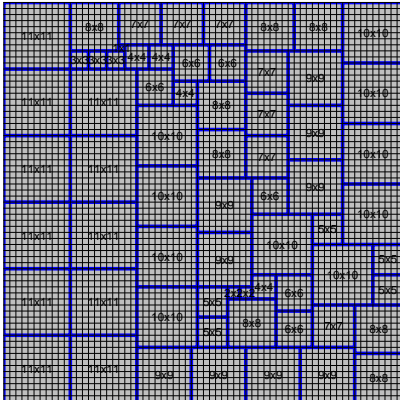
# Dominoes, Optimal Solution (N=22)



## Multiple items of same size

- Introduces additional symmetries
- Removing symmetry by lexicographic constraints
- Constrain all items of same size
- Works for square or rectangle items

# Partridge, Optimal Solution (N=11)





# Summary

- Rectangle packing problems can be solved by off-the-shelf constraint technology.
- Choice of search strategy of prime importance.
- Out-performs previous best solution by factor 1000.
  
- Future Research
  - Which impact has slack in the problem?
  - Faster heuristics for feasible problems?